Murison's Activities in the Dynamical Astronomy Division

Marc A. Murison

Astronomical Applications Dept.
U.S. Naval Observatory
November 6, 1996

Overview

- Optics (work begun while with the POINTS team at the Smithsonian Astrophysical Observatory)
  - RayTrace (numerical ray tracing)
  - AESOP (symbolic ray tracing)
  - FAME
- Dynamics (Chaotic Motion in the Solar System)
  - Outer asteroid belt – Lyapunov exponent relation
  - Dynamics from a Riemannian geometry perspective
  - Newcomb – a new Solar System ephemeris program

What is AESOP?

- AESOP = An Extensible Symbolic Optics Package
- AESOP is a suite of procedures, written in the Maple programming language, for performing *symbolic* ray tracing of optical systems.
- Uses geometric optics (i.e., no diffractive propagation).
- Useful for:
  - small optical element misalignments or other perturbations
  - ray and aperture-averaged path length variations
  - perturbed wavefront analysis
  - two or more simultaneous independent perturbations
  - developing physical intuition (esp. in pm regime)

How Does AESOP Work?

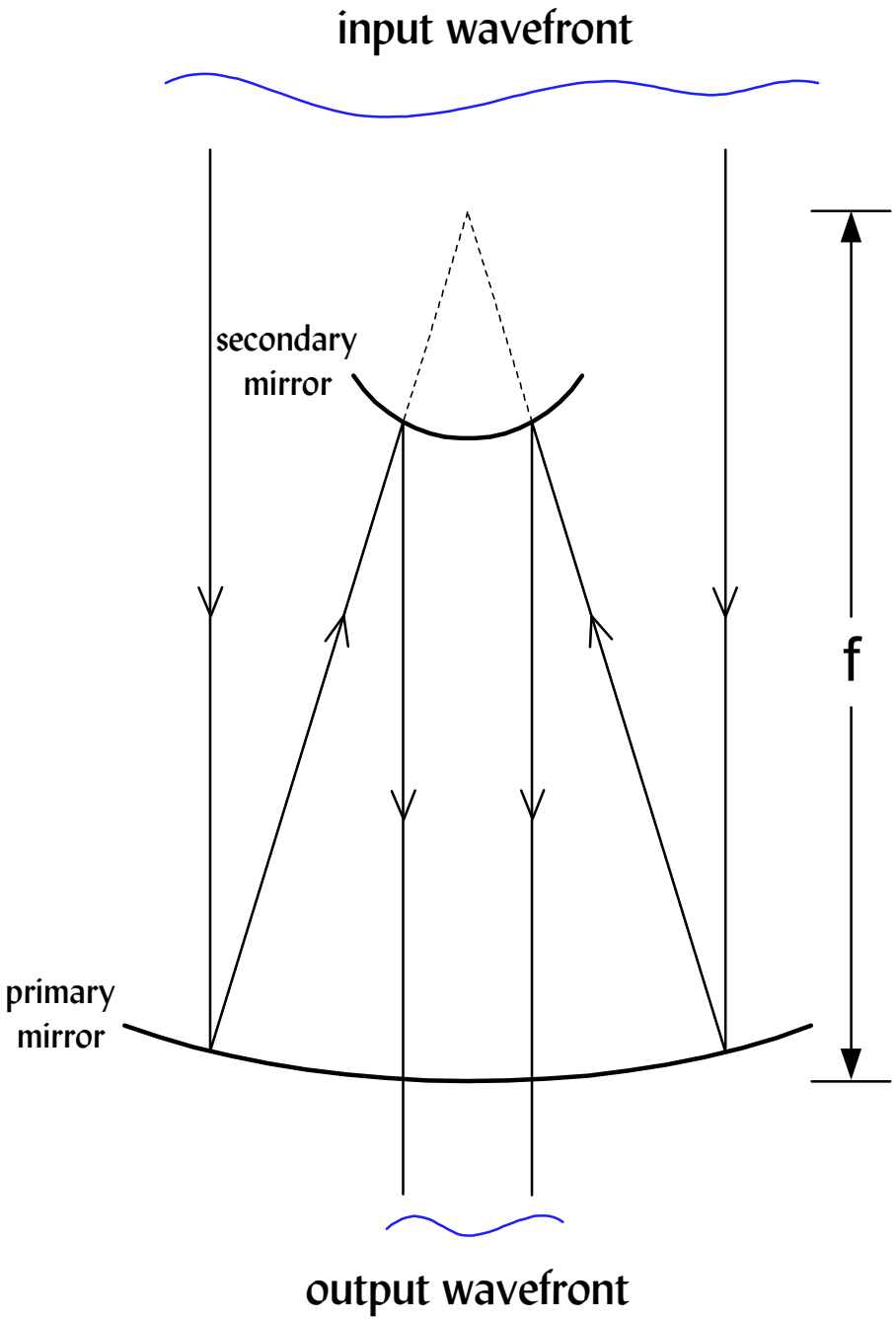
- Start with a geometric input ray
  - anchor point (previous intersection point)
  - direction
- Intersect with next optical surface
  - put $\vec{r} = \vec{p} + \vec{n}\ t$ into surface equation
  - solve for geometric path t
- Calculate accumulated optical path
- Calculate surface normal vector
- Calculate output ray direction
  - reflect
  - refract
  - diffract (HOE = Holographic Optical Element)

- Secret for success: series expansions at crucial points in the algebraic calculations
  - perturbation parameters
  - beam radius

How Does One Use AESOP?

- Define the optical surfaces $s(\rho) = \dfrac{\rho^2}{2f + \sqrt{4f^2 - \varepsilon\rho^2}}$
  - mirror
  - lens
  - HOE
- Perturb the optical elements
  - rotations
  - translations
  - surface shape modifications
- Define the input ray $r = p + n\,t$
- Call raytrace()
- Calculate the OPD and the aperture-averaged OPD
- Determine misalignment tolerances
- Analyze the wavefront
  - Zernike series expansion of wavefront
  - isolate individual aberration terms

Example: A Beam Compressor



Example: A Beam Compressor

- Perturbations:
  - lateral translation of the primary mirror
  - rotation of the primary mirror
- The OPD (i.e., the wavefront) at the detector is of the form

$$OPD = \sum_{n,m=0}^{n+m \le N_p} \psi^n \Delta^m \cdot \begin{cases} \sum_{k=0}^{n+m} \left[\cos^{2k}\varphi \cdot \sum_{j=2k}^{\frac{N_R}{2}} \left(\frac{A_{2j}(C,d)}{f^{2j}} + \frac{B_{2j}(C)}{f^{2j-1}} \right) \rho^{2j} \right] & n+m \text{ even} \\ \sum_{k=1}^{n+m} \left[\cos^{2k-1}\varphi \cdot \sum_{j=2k-1}^{\left[\frac{N_R+1}{2}\right]-1} \left(\frac{D_{2j-1}(C,d)}{f^{2j+2}} + \frac{E_{2j-1}(C)}{f^{2j+1}} \right) \rho^{2j-1} \right] & n+m \text{ odd} \end{cases}$$

- Aperture-averaged OPD simplifies greatly due to symmetry.
- Zernike representation of wavefront is convenient but complex:

```
> cost(S);
```
1605 additions + 11173 multiplications + 690 divisions + 17 functions

```
> cost(opd);
```
433 additions + 3710 multiplications + 187 divisions + 13 functions

Aperture-Averaged OPD

$$\langle OPD\rangle = \Delta^2 \cdot \left\{ \left[\frac{1}{2}\frac{C^2}{f^2} - \frac{1}{4}\frac{C^2\left(a^2+b^2\right)}{f^4} + \frac{11}{96}\frac{C^2\left(b^2-ab+a^2\right)\left(ab+b^2+a^2\right)}{f^6} \right.\right.$$

$$- \frac{3}{64}\frac{C^2\left(a^2+b^2\right)\left(a^4+b^4\right)}{f^8}$$

$$+ \frac{9}{512}\frac{C^2\left(b^4-b^3a+b^2a^2-ba^3+a^4\right)\left(b^3a+b^2a^2+b^4+ba^3+a^4\right)}{f^{10}}$$

$$- \frac{19}{3072}\frac{C^2\left(a^2+b^2\right)\left(b^4-b^2a^2+a^4\right)\left(b^2-ab+a^2\right)\left(ab+b^2+a^2\right)}{f^{12}}$$

$$\left.\left. + \frac{17}{8192}\frac{C^2\left(a^{12}+b^{12}+a^6b^6+a^8b^4+a^{10}b^2+b^{10}a^2+b^8a^4\right)}{f^{14}} \right] \cdot d \right.$$

$$+ \frac{-1+\frac{1}{2}C^2}{f} - \frac{1}{8}\frac{\left(a^2+b^2\right)\left(-C+2C^2-2\right)}{f^3}$$

$$+ \frac{1}{96}\frac{\left(b^2-ab+a^2\right)\left(ab+b^2+a^2\right)\left(11C^2-6-8C\right)}{f^5}$$

$$- \frac{1}{256}\frac{\left(a^2+b^2\right)\left(a^4+b^4\right)\left(-4-11C+12C^2\right)}{f^7}$$

$$+ \frac{1}{2560}\frac{\left(a^8+b^8+b^6a^2+b^4a^4+b^2a^6\right)\left(-10-48C+45C^2\right)}{f^9}$$

$$- \frac{1}{3072}\frac{\left(a^2+b^2\right)\left(b^4-b^2a^2+a^4\right)\left(b^2-ab+a^2\right)\left(ab+b^2+a^2\right)\left(19C^2-3-25C\right)}{f^{11}}$$

$$\left. + \frac{1}{57344}\frac{\left(a^{12}+b^{12}+a^6b^6+a^8b^4+a^{10}b^2+b^{10}a^2+b^8a^4\right)\left(-14+119C^2-174C\right)}{f^{13}} \right\}$$

Classical Coma Term

$$
\begin{aligned}
OPD_{coma} = \; & \left(3\rho^3 - 2\rho\right)\cos\phi \cdot \Bigg\{ \\
& \Delta^3 \cdot \Bigg[\left(\frac{5}{57344}\,\frac{C^2(-1106+1259\,C)d}{f^{14}} + \frac{5}{57344}\,\frac{1259\,C^4-3212\,C^3-7\,C+70+1611\,C^2}{f^{13}C} \right) R^{11} \\
& \quad + \left(-\frac{1}{1344}\,\frac{C^2(-295+286\,C)d}{f^{12}} - \frac{1}{2688}\,\frac{770\,C^2-1483\,C^3+572\,C^4-25\,C+70}{f^{11}C} \right) R^9 \\
& \quad + \left(\frac{1}{320}\,\frac{C^2(-140+111\,C)d}{f^{10}} + \frac{1}{320}\,\frac{146\,C+15+111\,C^3-286\,C^2}{f^9} \right) R^7 \\
& \quad + \left(-\frac{7}{80}\,\frac{C^2(-8+5\,C)d}{f^8} - \frac{1}{80}\,\frac{35\,C^3+49\,C-99\,C^2+9}{f^7} \right) R^5 \\
& \quad + \left(\frac{1}{24}\,\frac{C^2(-17+8\,C)d}{f^6} + \frac{1}{48}\,\frac{(4\,C+1)(4\,C^2-14\,C+9)}{f^5} \right) R^3 \Bigg] \\
& + \psi\Delta^2 \cdot \Bigg[\left(\frac{15}{28672}\,\frac{C^2(743\,C-475)d}{f^{13}} + \frac{5}{57344}\,\frac{-10342\,C^3+4458\,C^4+4290\,C^2+64\,C-65}{f^{12}C} \right) R^{11} \\
& \quad + \left(-\frac{1}{1344}\,\frac{C^2(1063\,C-825)d}{f^{11}} - \frac{1}{2688}\,\frac{2244\,C^2-4992\,C^3+72\,C+2126\,C^4-65}{f^{10}C} \right) R^9 \\
& \quad + \left(\frac{9}{320}\,\frac{C^2(49\,C-48)d}{f^9} + \frac{3}{320}\,\frac{2-342\,C^2+147\,C^3+164\,C}{f^8} \right) R^7 \\
& \quad + \left(-\frac{3}{80}\,\frac{C^2(51\,C-65)d}{f^7} - \frac{1}{80}\,\frac{7+191\,C+153\,C^3-388\,C^2}{f^6} \right) R^5 \\
& \quad + \left(\frac{1}{24}\,\frac{C^2(40\,C-69)d}{f^5} + \frac{1}{48}\,\frac{80\,C^3-230\,C^2+104\,C+13}{f^4} \right) R^3 \Bigg] \\
& + \psi^2\Delta \cdot \Bigg[\left(\frac{25}{57344}\,\frac{C^2(-420+1039\,C)d}{f^{12}} + \frac{5}{57344}\,\frac{-12\,C-65-10868\,C^3+5195\,C^4+3329\,C^2}{f^{11}C} \right) R^{11} \\
& \quad + \left(-\frac{1}{1344}\,\frac{C^2(1291\,C-683)d}{f^{10}} - \frac{1}{2688}\,\frac{-5453\,C^3+11\,C-65+1962\,C^2+2582\,C^4}{f^9C} \right) R^9 \\
& \quad + \left(\frac{1}{320}\,\frac{C^2(567\,C-404)d}{f^8} + \frac{1}{320}\,\frac{567\,C^3-17+507\,C-1185\,C^2}{f^7} \right) R^7 \\
& \quad + \left(-\frac{1}{80}\,\frac{C^2(-209+214\,C)d}{f^6} - \frac{1}{160}\,\frac{460\,C+428\,C^3-17-974\,C^2}{f^5} \right) R^5 \\
& \quad + \left(\frac{1}{3}\,\frac{C^2(8\,C-11)d}{f^4} + \frac{1}{48}\,\frac{-3+128\,C^3-328\,C^2+156\,C}{f^3} \right) R^3 \Bigg] \\
& + \psi^3 \cdot \Bigg[\left(\frac{5}{14336}\,\frac{C^2(-86+499\,C)d}{f^{11}} + \frac{5}{172032}\,\frac{-11178\,C^3+17+1890\,C^2+5988\,C^4}{f^{10}C} \right) R^{11} \\
& \quad + \left(-\frac{1}{672}\,\frac{C^2(-74+257\,C)d}{f^9} - \frac{1}{8064}\,\frac{3084\,C^4+1416\,C^2-5802\,C^3+17-21\,C}{f^8C} \right) R^9 \\
& \quad + \left(\frac{3}{320}\,\frac{C^2(-36+79\,C)d}{f^7} + \frac{1}{960}\,\frac{711\,C^3-8-1323\,C^2+465\,C}{f^6} \right) R^7 \\
& \quad + \left(-\frac{1}{80}\,\frac{C^2(96\,C-67)d}{f^5} - \frac{1}{480}\,\frac{-19+504\,C-1170\,C^2+576\,C^3}{f^4} \right) R^5 \\
& \quad + \left(\frac{1}{12}\,\frac{C^2(-17+16\,C)d}{f^3} + \frac{1}{144}\,\frac{-444\,C^2+216\,C+192\,C^3-7}{f^2} \right) R^3 \Bigg] \\
& + \left(\psi f + \Delta\right) \cdot \left[-\frac{5}{14336}\,\frac{R^{11}}{f^{11}} + \frac{1}{672}\,\frac{R^9}{f^9} - \frac{1}{160}\,\frac{R^7}{f^7} + \frac{1}{40}\,\frac{R^5}{f^5} - \frac{1}{12}\,\frac{R^3}{f^3} \right] \Bigg\}
\end{aligned}
$$

Example: A Beam Compressor

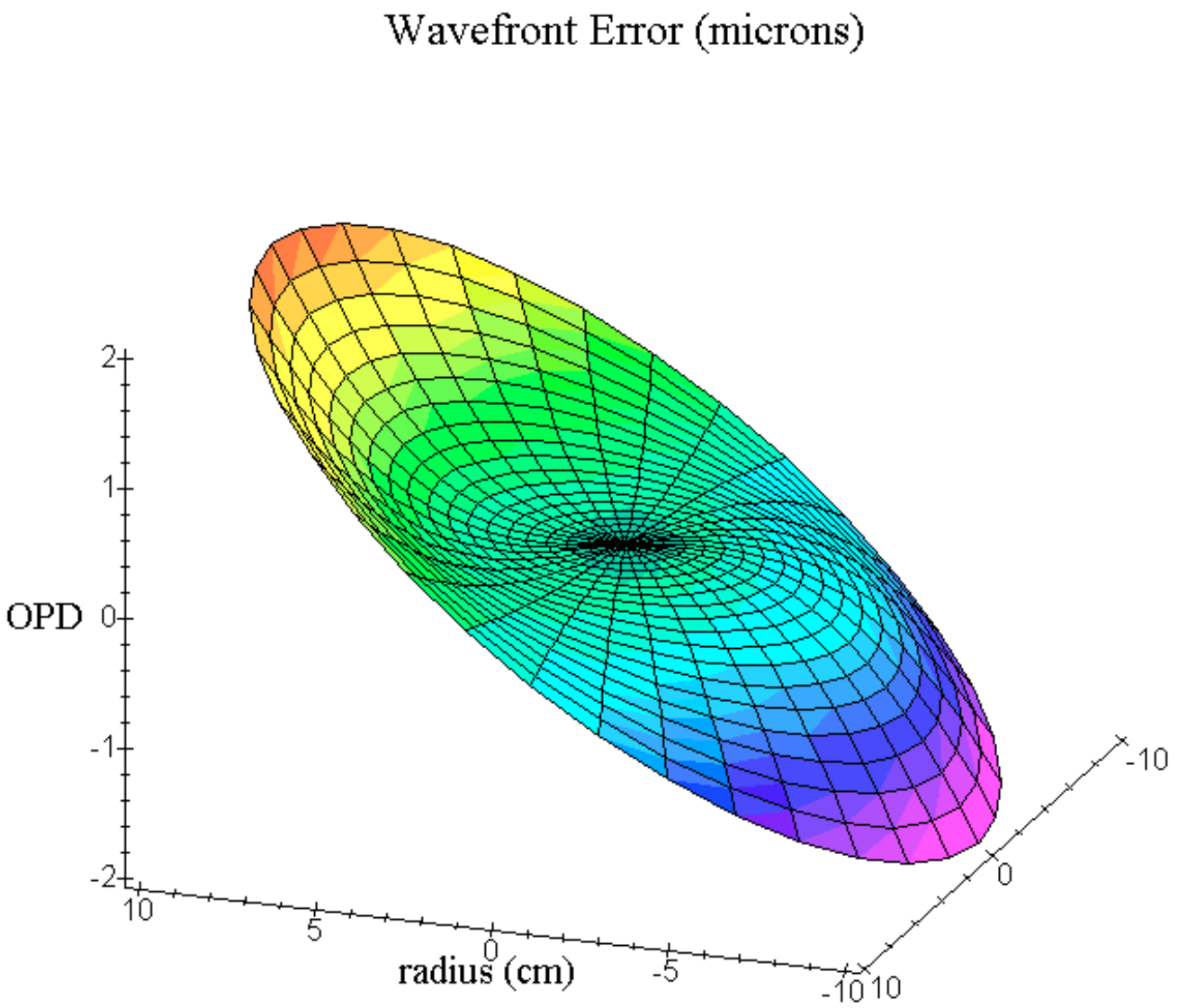
- Suppose we have just a translation. How much should we rotate the primary to compensate for the resulting coma?

$$\psi = -\frac{\Delta}{f} - \frac{\begin{array}{l}\left[75C^3(3199C-1744)d + 75f\left(71C+2679C^2+3199C^4-135-7130C^3\right)\right]R^8 \\ +\left[-640C^3f^2(-530+777C)d - 320f^3\left(-3509C^3+1474C^2+97C-135+1554C^4\right)\right]R^6 \\ +\left[5376C^3f^4(-146+165C)d + 2688f^5C\left(-740C^2+346C-9+330C^3\right)\right]R^4 \\ +\left[-10752C^3f^6(-139+118C)d - 10752f^7C\left(118C^3-289C^2+142C-2\right)\right]R^2 \\ +143360C^3f^8(-13+8C)d + 35840f^9C\left(41C-89C^2+2+32C^3\right)\end{array}}{4Cf\left(-320R^6f^2+75R^8-5376R^2f^6+17920f^8+1344R^4f^4\right)} \cdot \frac{\Delta^3}{f^3}$$

- First order term is as expected
- Next correction term is cubic
- Graphic illustration:

Aberrated Wavefront from a Misaligned Beam Compressor

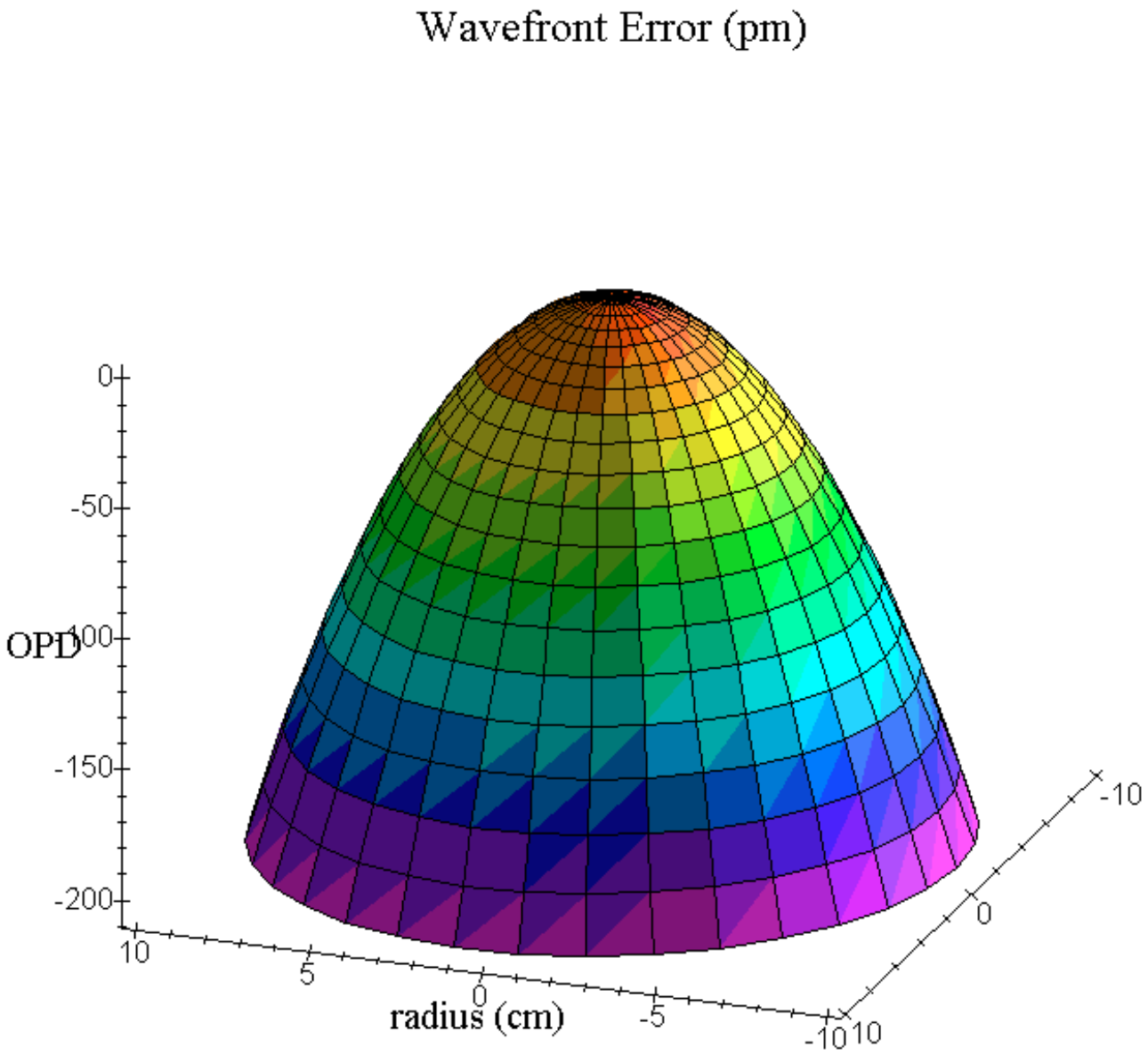
- Lateral translation of primary by 10 microns...



- ...results in ~2 microns of coma at edge of output beam
- This is bad.

Aberrated Wavefront from a Misaligned Beam Compressor

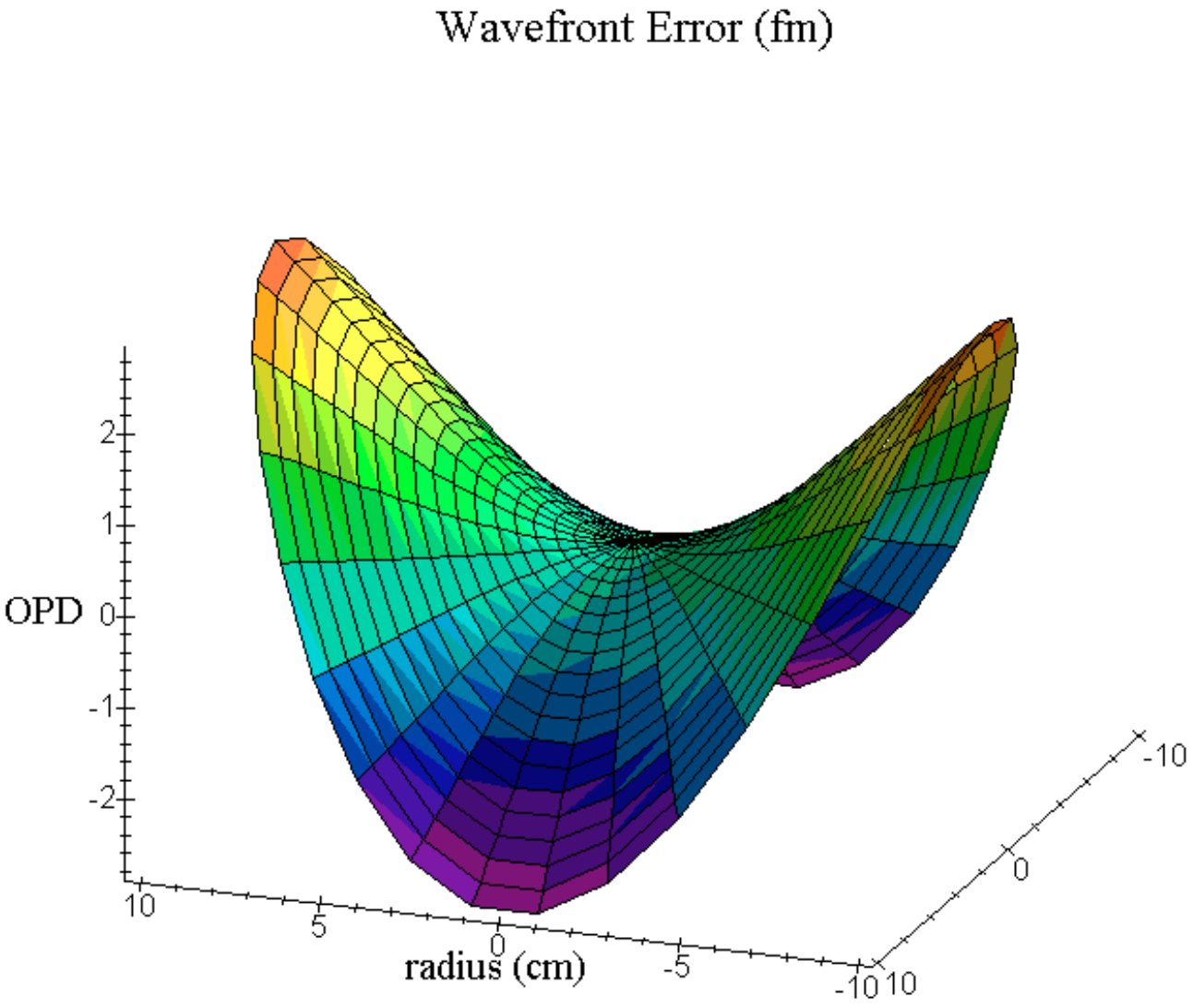
- Compensate by rotating the primary by the magic angle:



- What's left: ~200 *picometers* (pm) of defocus at edge of output beam. Residual coma is ~3 pm.

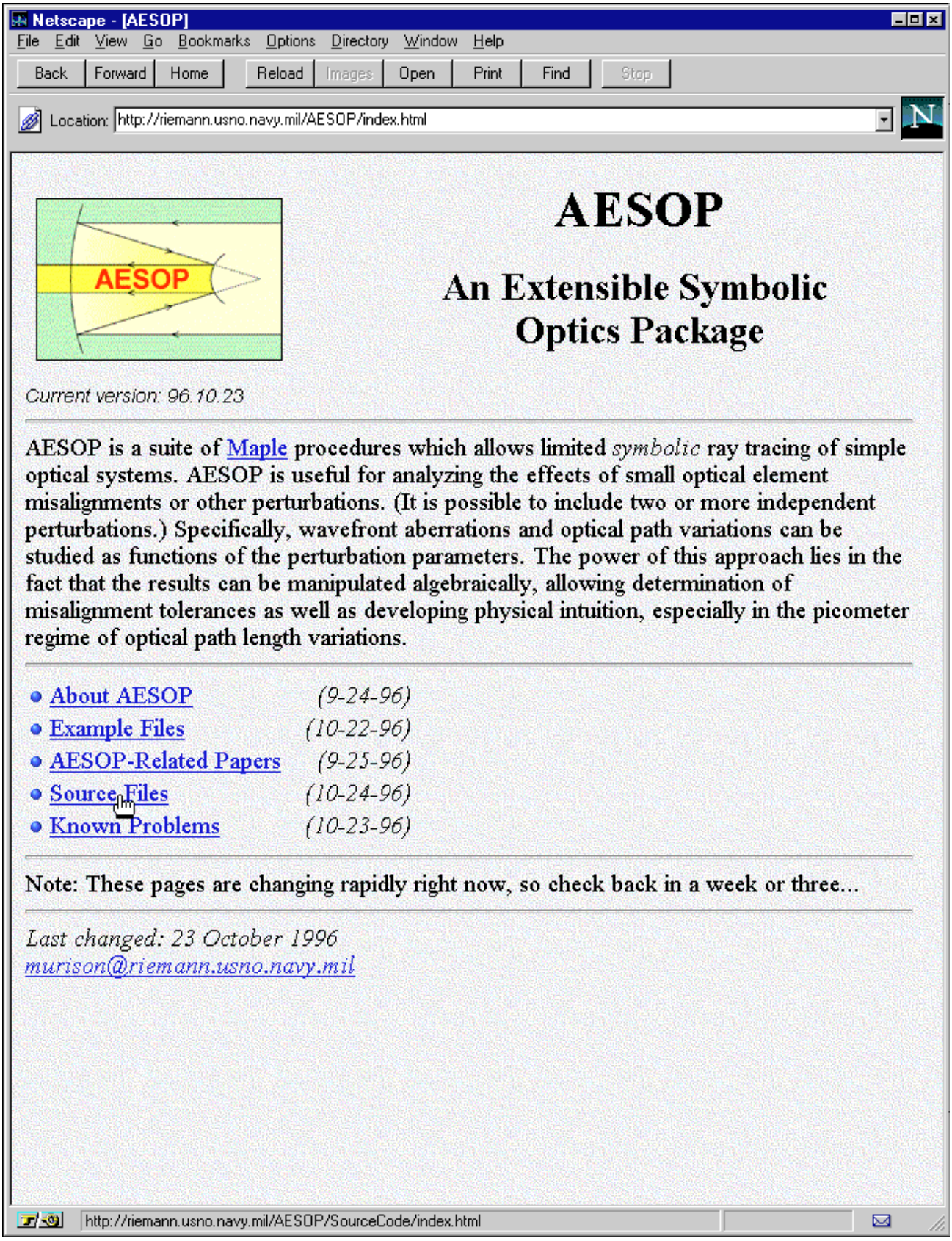
Aberrated Wavefront from a Misaligned Beam Compressor

- Is there anything else? Let's remove all coma and defocus terms:

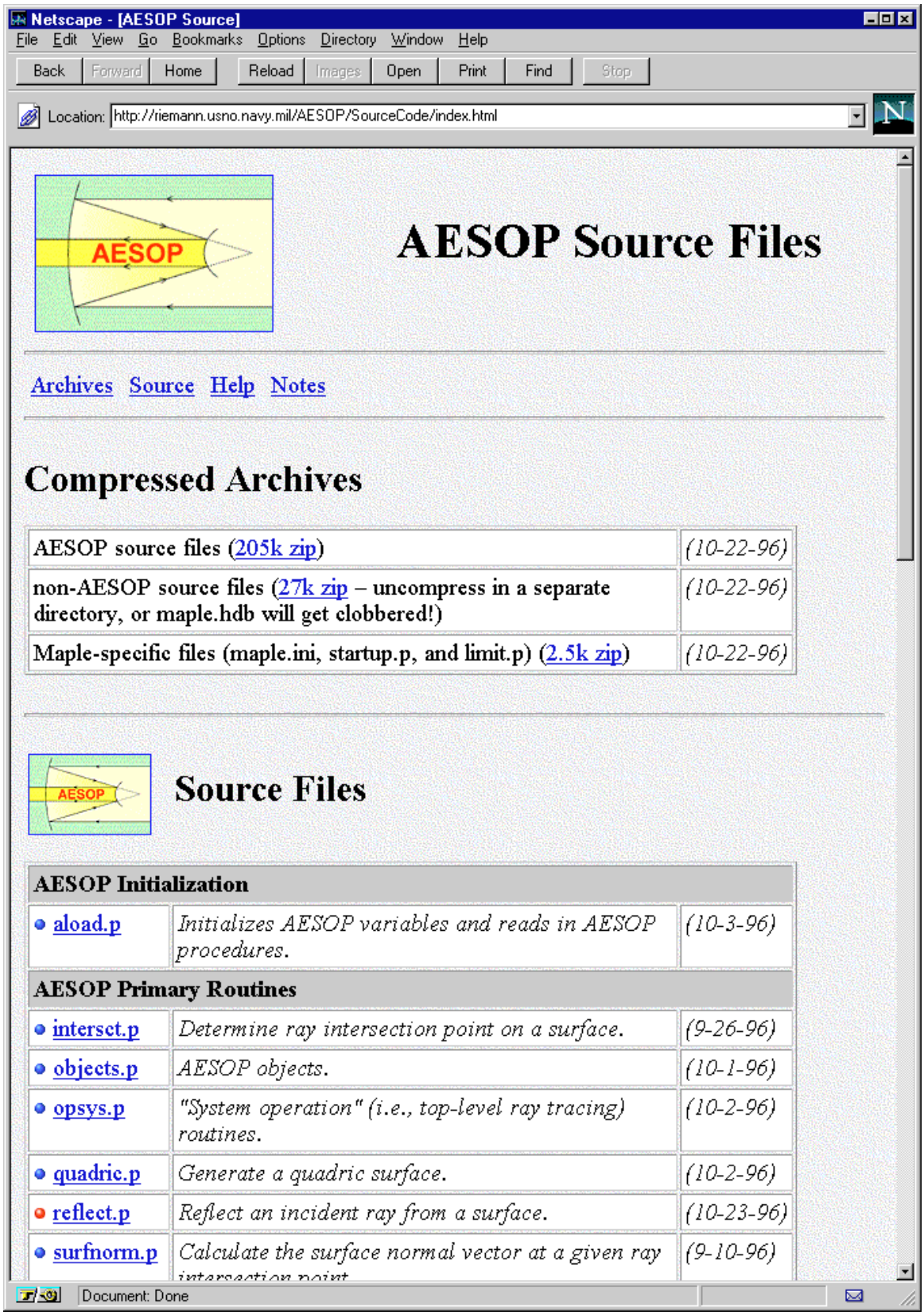


- Residual is ~3 *femtometers* of astigmatism.

AESOP on the Web: Home Page



AESOP on the Web: Source Files Page



Riemannian Dynamics

- Chaotic dynamics from a Riemannian geometry perspective.
  - Language of GR, but not looking specifically at GR.
- Ideas first explored by Krylov (1950) studying gas relaxation; then by Gurzadyan and Savvidy (1984, 1986) with relaxation of stellar systems. Recent work by Casetti et al. (1990-1996) on N-body problem (large N).
- Background:
  - Newtonian eqs. of motion

$$\frac{d^2 q^i}{dt^2} + \frac{\partial V(q)}{\partial q^i} = 0$$

  - Resulting variational equations

$$\frac{d^2 \xi^i}{dt^2} + \frac{\partial^2 V(q)}{\partial q^i \partial q^k} \xi^k = 0$$

    - "spread" of trajectories
    - stability

Riemannian Dynamics

- Similarly, geodesic equations of motion

$$\frac{d^2 q^i}{ds^2} + \Gamma^i_{jk} \frac{dq^j}{ds} \frac{dq^k}{ds} = 0$$

- Resulting equations of geodesic spread – the Jacobi equation

$$\frac{D^2 J^i}{Ds^2} + R^i_{jkm} \frac{dq^j}{ds} \frac{dq^m}{ds} J^k = 0, \qquad J = \frac{d\gamma(s,\tau)}{d\tau}$$

$$where \quad \frac{DJ^i}{Ds} = \frac{dJ^i}{ds} + \Gamma^i_{jk} \frac{dq^j}{ds} J^k$$

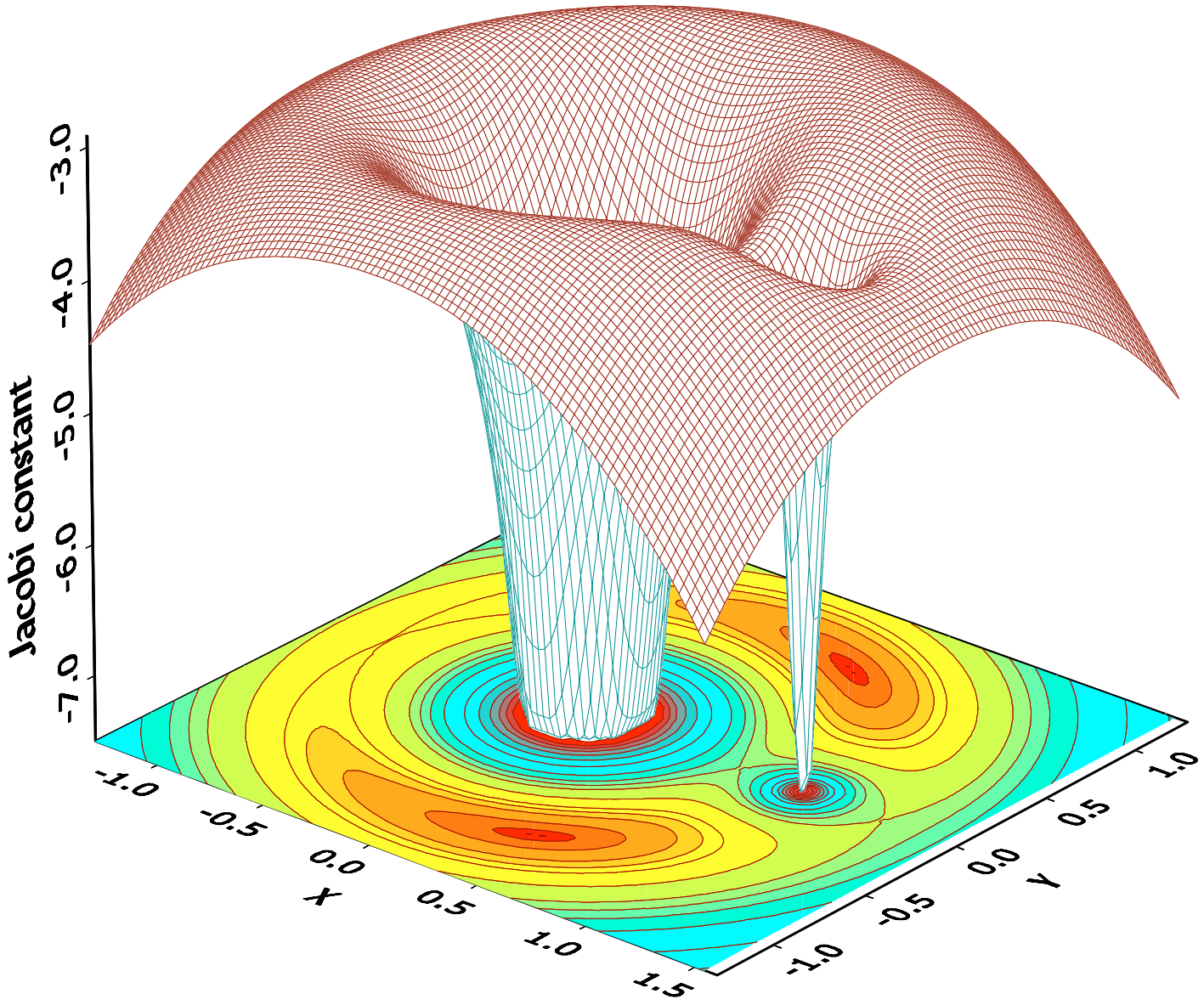
- With the metric $g_{jk} = (E - V)\delta_{jk}$ the geometric equations are equivalent to the Newtonian equations if the arc length is written $ds = \sqrt{2}(E - V)dt$

• Research Directions

- Start with restricted three-body problem (CRTB, ERTB).
- Calculate evolution of Jacobi field J for CRTB orbits.
  - Numerical
  - Analytical
- Concentrate on "crossover" orbits.
- Relate to Lyapunov exponent, resonance features, stability.

CRTB Energy Manifold

$Z = 0 \quad \mu = 0.1$



Newcomb – A New Ephemeris Program

- Why a new ephemeris program?
  - Develop USNO self-sufficiency and in-house expertise.
    - USNO publishes the AA.
    - JPL retirements looming...
    - Ephemeris program of this type will never be a "black box".
    - Hence, the only way to become proficient is to write our own program.
  - Independent check on JPL program and PEP.
    - It is quite unusual to have only 2 programs available upon which an entire field depends.
  - Existing software getting mighty creaky.
    - inflexible
    - maintenance nightmare
    - unfriendly and unforgiving
    - not portable

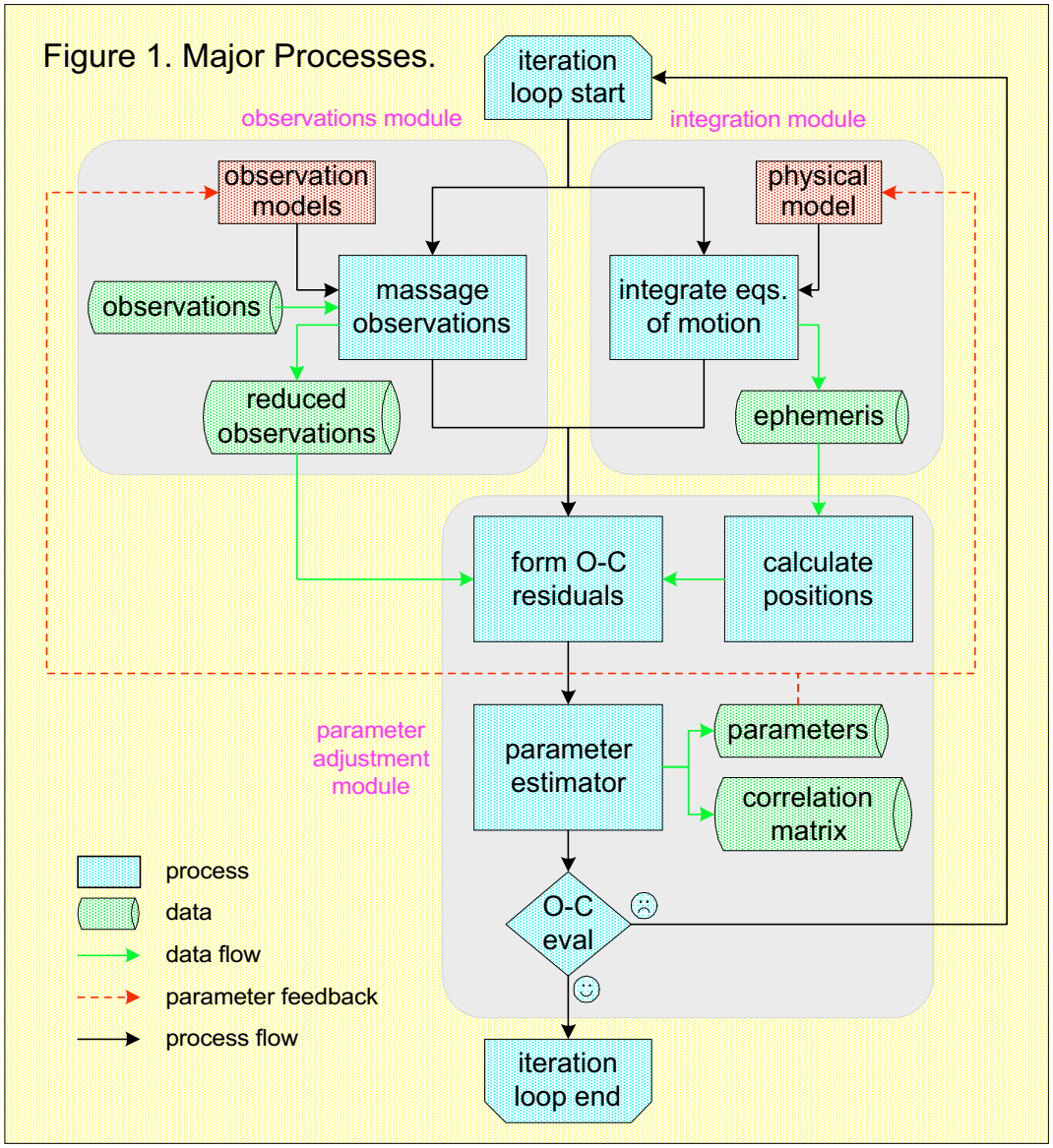
Newcomb – A New Ephemeris Program

- Why a new ephemeris program? (continued)
  - An object-oriented design address all of the major software problems.
    - A more natural description of real-world problems leads to programs that are easier to write, understand, and maintain.
    - Sophistication of tasks one can tackle is at least an order of magnitude higher.
    - Maintenance costs are an order of magnitude less.
    - OOPs are flexible and extensible.
    - A decent GUI becomes feasible.
    - Portable!

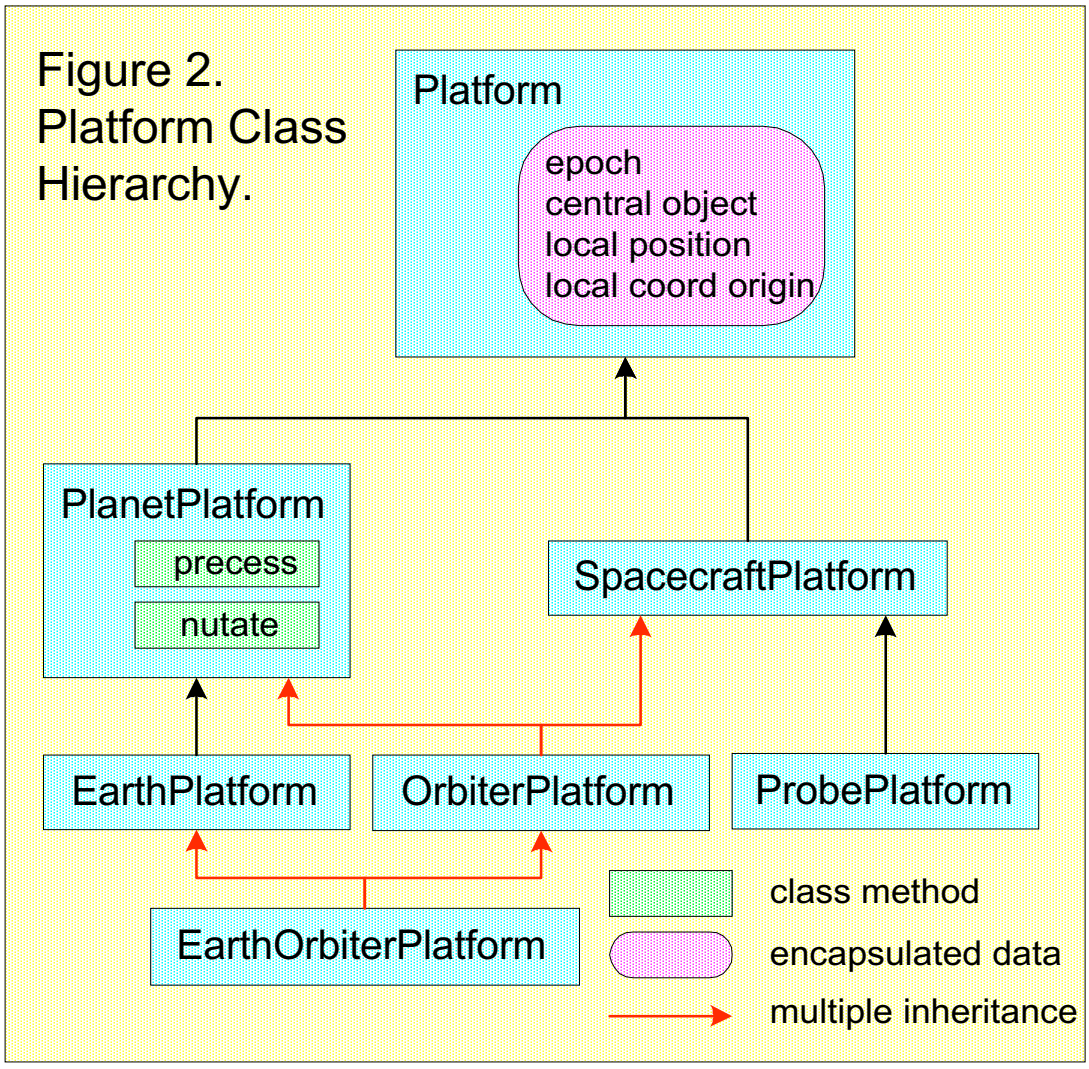
Newcomb – A New Ephemeris Program

- Science
  - dG/dt
  - Asteroids
    - PEP: 15 self-consistent objects
    - JPL: 10 self-consistent objects
    - masses, finally
    - effects on Mars ephemeris
  - Timescale comparisons
  - Discrepancy between inner & outer solar system ephemerides
- Structure overview
- Interface

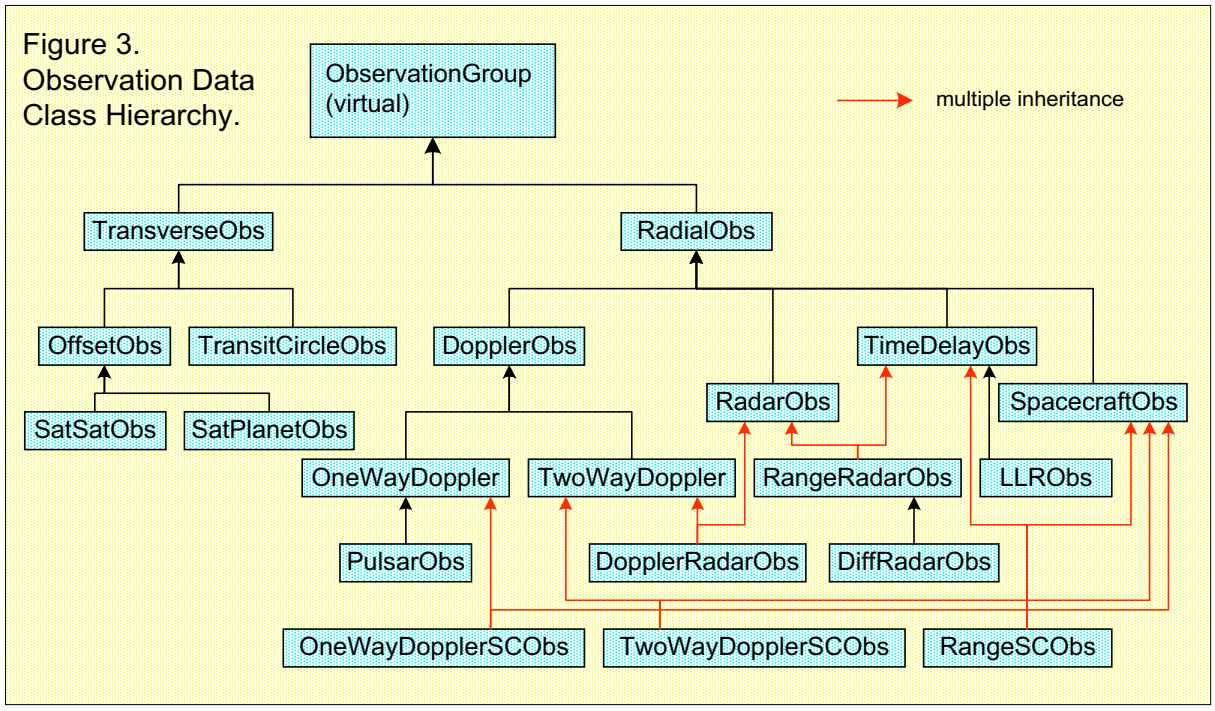
Newcomb Process Overview



Newcomb Platform Class Hierarchy



Newcomb Datatype Hierarchy



Newcomb Parameter Adjustment Overview

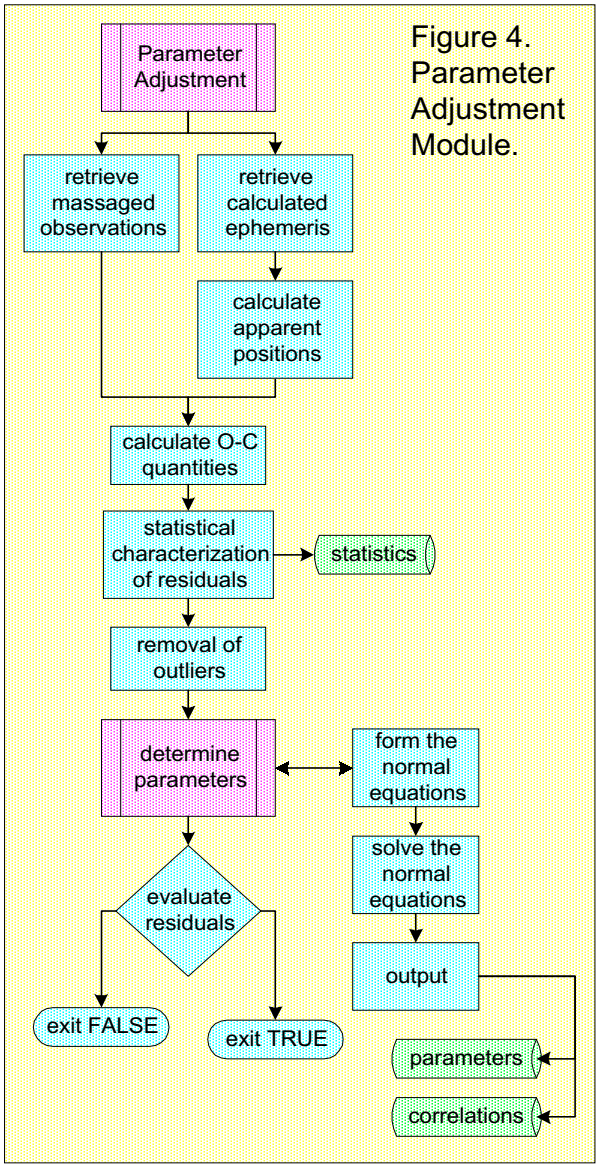
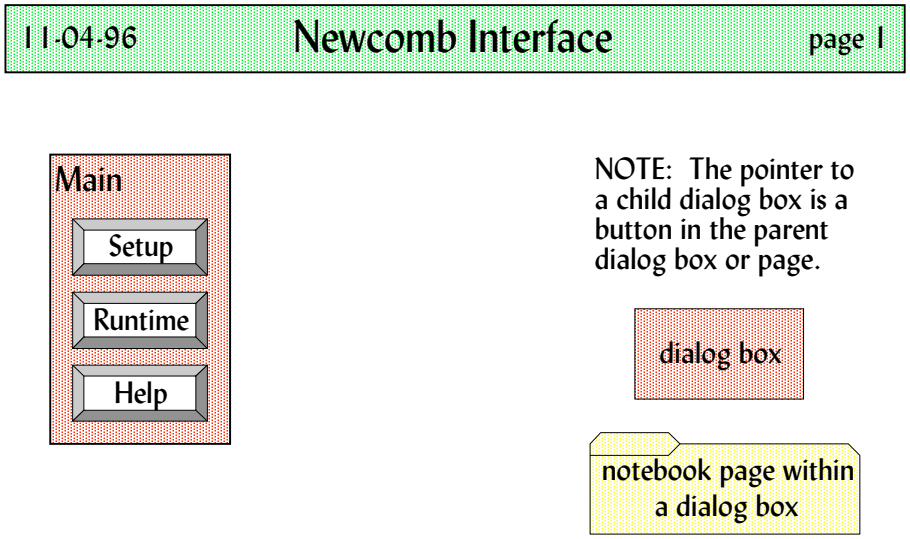
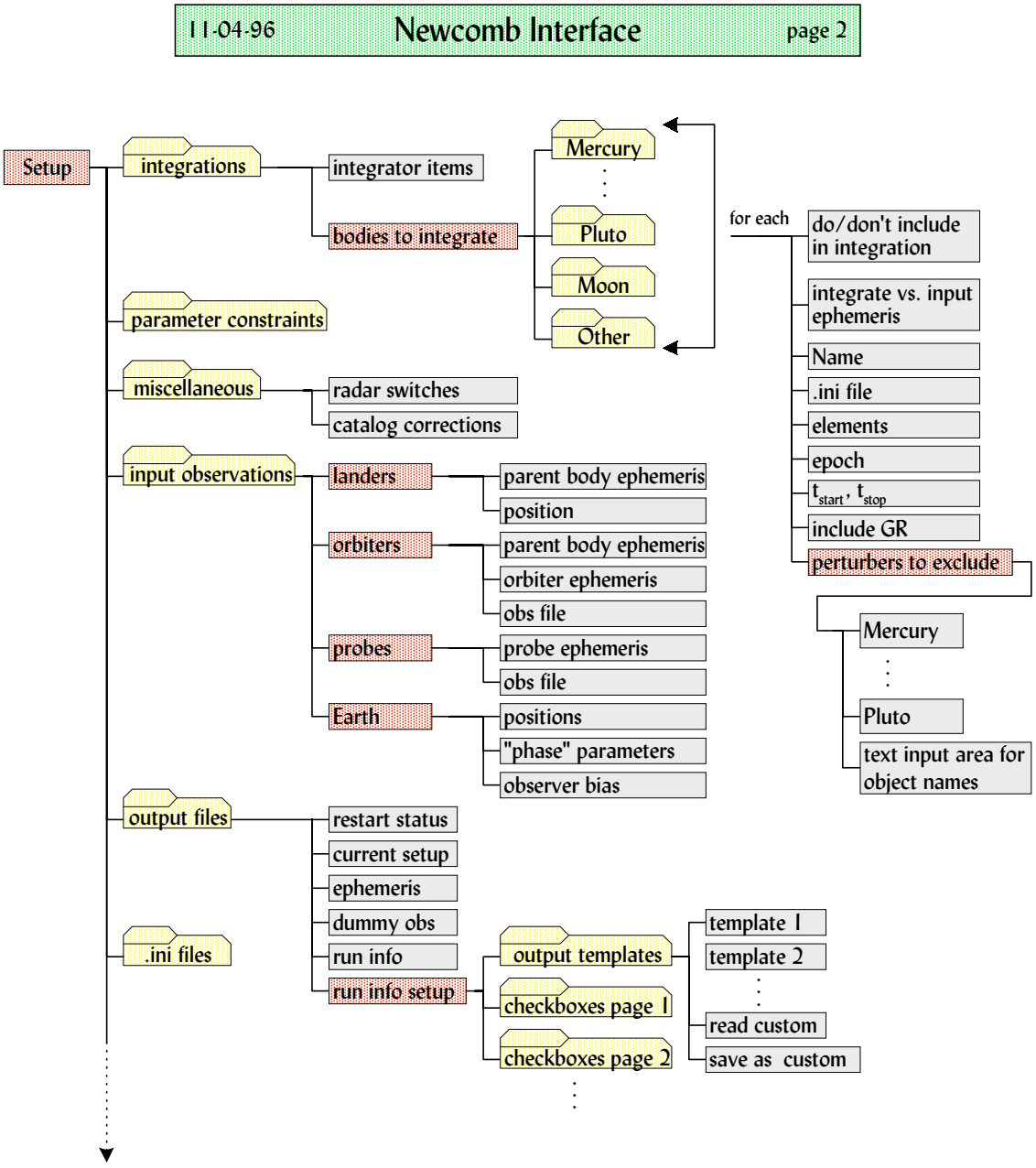


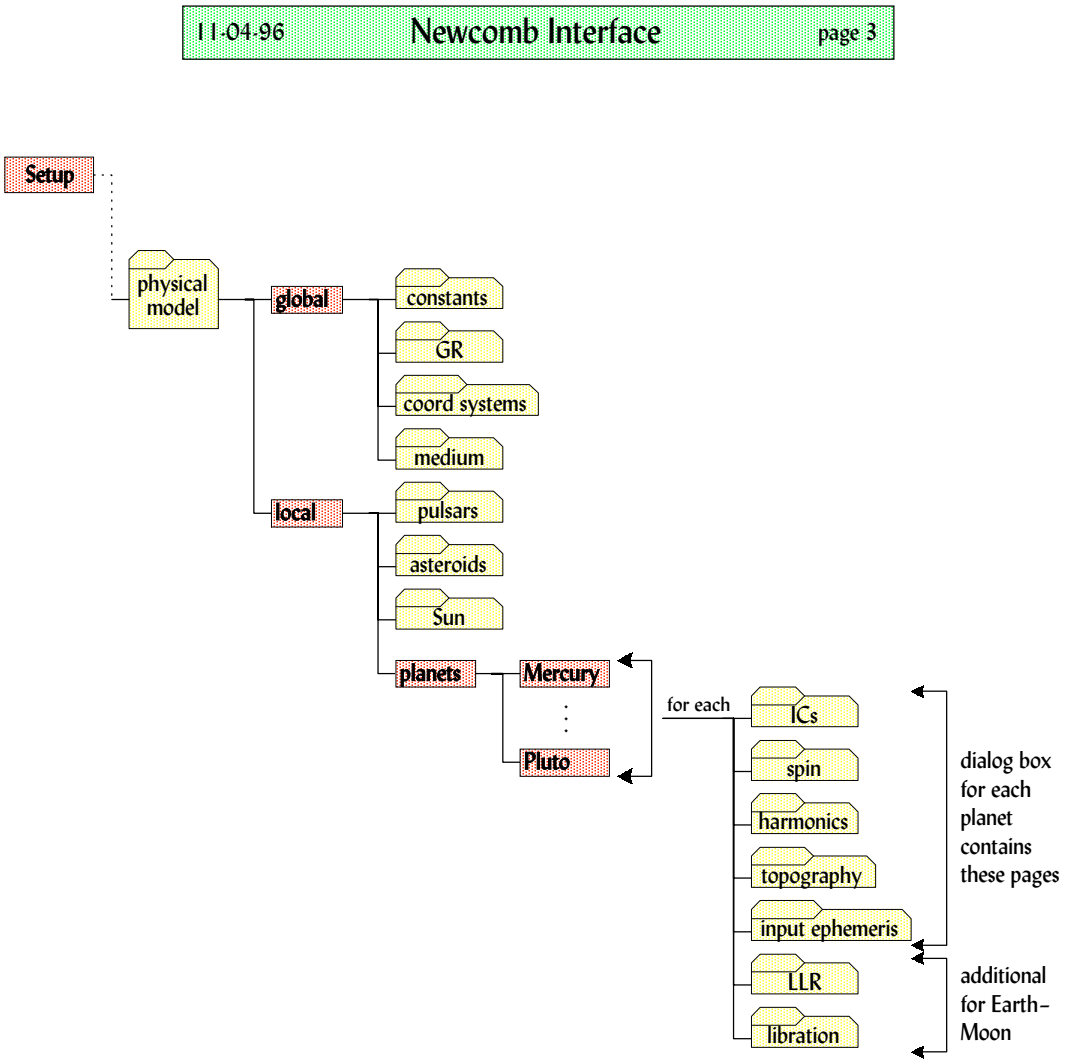
Figure 4. Parameter Adjustment Module.


11-04-96
Newcomb Interface
page 1
Main
Setup
Runtime
Help
NOTE: The pointer to a child dialog box is a button in the parent dialog box or page.
dialog box
notebook page within a dialog box
Runtime
operation control
stop/go
least squares on/off
partials: compute or read from file
convergence criteria
data reject criteria
current status
iteration
process
info relevant to current process
convergence
view partials

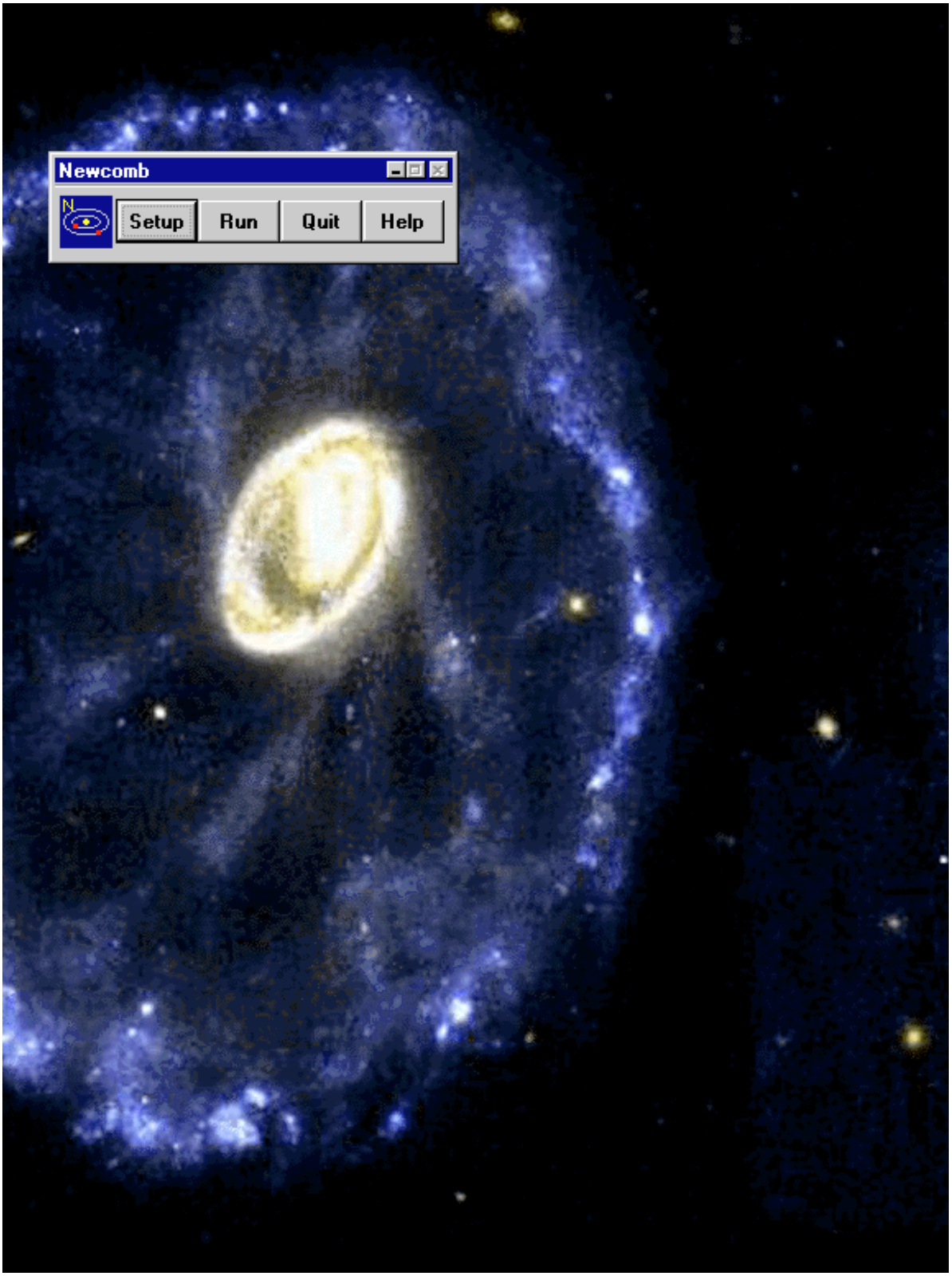
Newcomb GUI


11-04-96
Newcomb Interface
page 2
Setup
integrations
integrator items
bodies to integrate
Mercury
Pluto
Moon
Other
for each
do/don't include in integration
integrate vs. input ephemeris
Name
.ini file
elements
epoch
t_{start}, t_{stop}
include GR
perturbers to exclude
Mercury
Pluto
text input area for object names
parameter constraints
miscellaneous
radar switches
catalog corrections
input observations
landers
parent body ephemeris
position
orbiters
parent body ephemeris
orbiter ephemeris
obs file
probes
probe ephemeris
obs file
Earth
positions
"phase" parameters
observer bias
output files
restart status
current setup
ephemeris
dummy obs
run info
run info setup
.ini files
output templates
checkboxes page 1
checkboxes page 2
template 1
template 2
read custom
save as custom

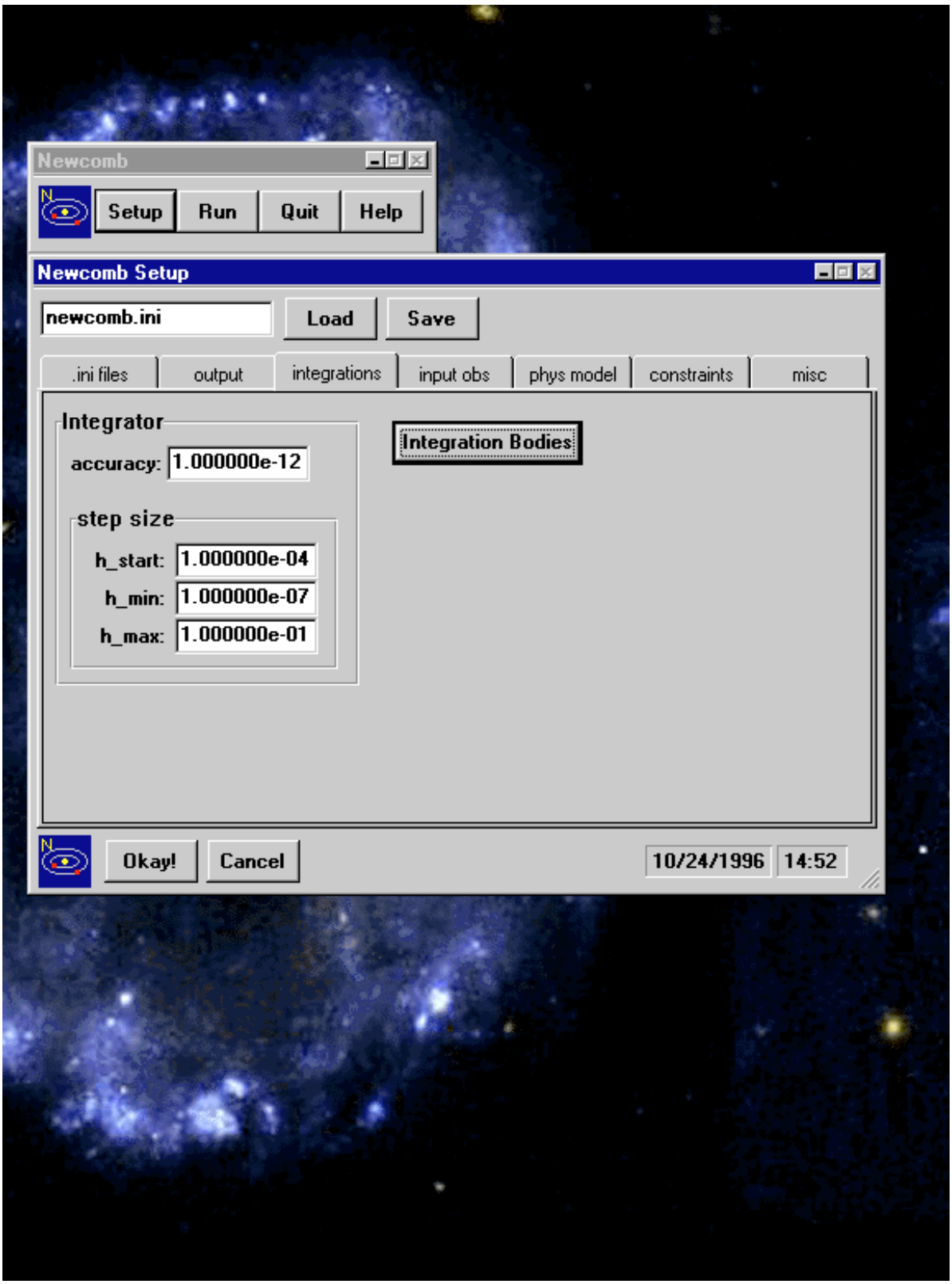
Newcomb GUI


11-04-96
Newcomb Interface
page 3
Setup
physical model
global
constants
GR
coord systems
medium
local
pulsars
asteroids
Sun
planets
Mercury
Pluto
for each
ICs
spin
harmonics
topography
input ephemeris
LLR
libration
dialog box for each planet contains these pages
additional for Earth-Moon

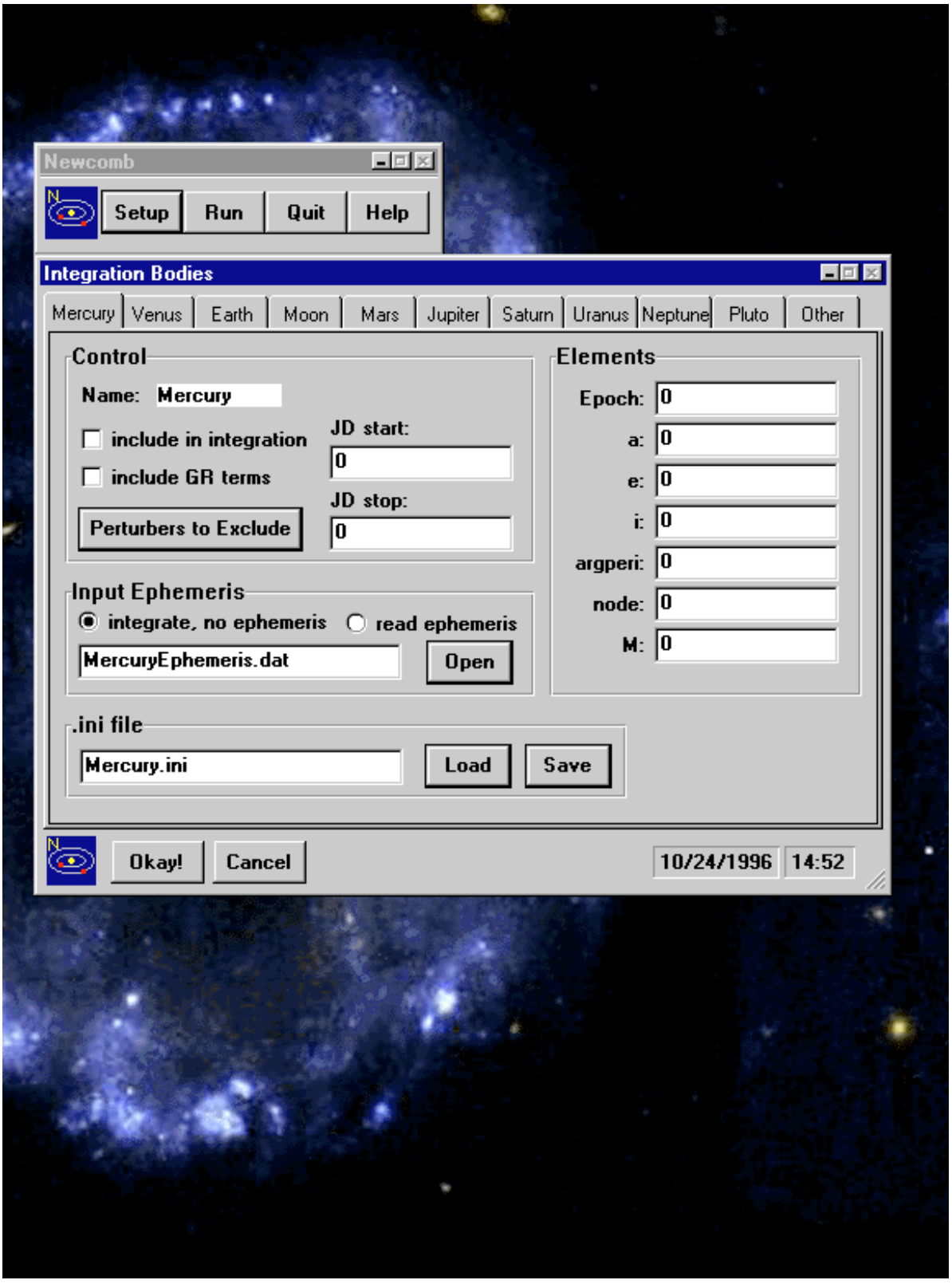
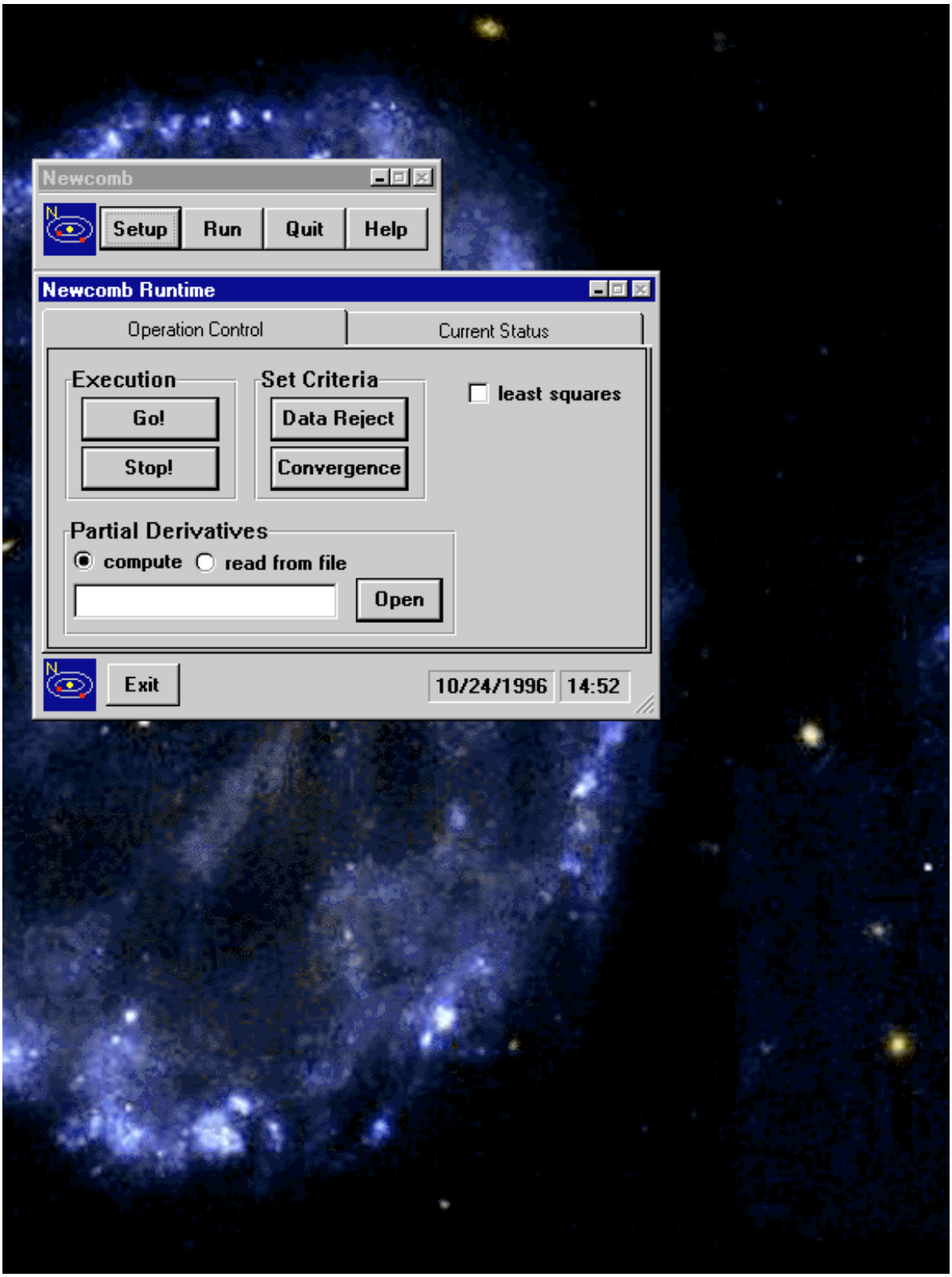
Newcomb Screenshots — Main Dialog



Newcomb Screenshots — Setup & Main



Newcomb Screenshots — Integration Bodies



Newcomb Screenshots — Runtime & Main